Journal of Statistical Mechanics:
Theory and Experiment

PAPER: INTERDISCIPLINARY STATISTICAL MECHANICS

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To cite this article: M Mazzoli and A Sánchez *J. Stat. Mech.* (2017) 113403

Journal of Statistical Mechanics: Theory and Experiment
An IOP and SISSA journal

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Equilibria, information and frustration in heterogeneous network games with conflicting preferences

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Received 13 June 2017
Accepted for publication 4 October 2017
Published 17 November 2017

Online at stacks.iop.org/JSTAT/2017/113403
https://doi.org/10.1088/1742-5468/aa9347

Abstract. Interactions between people are the basis on which the structure of our society arises as a complex system and, at the same time, are the starting point of any physical description of it. In the last few years, much theoretical research has addressed this issue by combining the physics of complex networks with a description of interactions in terms of evolutionary game theory. We here take this research a step further by introducing a most salient societal factor such as the individuals' preferences, a characteristic that is key to understanding much of the social phenomenology these days. We consider a heterogeneous, agent-based model in which agents interact strategically with their neighbors, but their preferences and payoffs for the possible actions differ. We study how such a heterogeneous network behaves under evolutionary dynamics and different strategic interactions, namely coordination games and best shot games. With this model we study the emergence of the equilibria predicted analytically in random graphs under best response dynamics, and we

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extend this test to unexplored contexts like proportional imitation and scale free networks. We show that some theoretically predicted equilibria do not arise in simulations with incomplete information, and we demonstrate the importance of the graph topology and the payoff function parameters for some games. Finally, we discuss our results with the available experimental evidence on coordination games, showing that our model agrees better with the experiment than standard economic theories, and draw hints as to how to maximize social efficiency in situations of conflicting preferences.

Keywords: evolutionary game theory, socio-economic networks, agent-based models, critical phenomena of socio-economic systems

Contents

1. Introduction

The behavior of complex systems is determined by their components and, chiefly, by their interactions. Generally speaking, specifying the interactions of a complex system [1] involves a network that indicates who interacts with whom, and the rule or law governing the interaction itself. This paradigm applies to purely physical systems, but

also to social systems [2, 3], the difference being that in the latter case interactions are strategic, i.e. the agents have some degree of intelligence and can anticipate the reactions of their counterparts to their own actions. Such a situation requires a description in terms of game theory [4] and, in fact, this framework is becoming very common and useful to describe complex systems in social and economic systems [3, 5–8]. However traditional game theory fails to explain which Nash equilibrium is to be selected when more than one equivalent choices are present [9]. Evolutionary game theory and agents based modeling give a solution to this problem: evolution selects the successful strategies, driving the population to an equilibrium while, on the other hand, agent based modeling tries to understand the results of adaptive behaviours in terms of emergence and self-organisation [10].

The complex systems community has devoted a lot of effort to this approach in this century (see, e.g. [11–14] for reviews). Typically, the models considered in this research are a combination of the above mentioned ingredients of games (describing how interactions take place) and networks (describing the interaction structure) with some evolutionary dynamics [15]. The rationale for such an approach is twofold: on one hand, several of the dynamics can be shown to lead to equilibrium states that are related to the Nash equilibria of the network game [4, 16], i.e. to what the system should be actually doing were it formed by rational agents. On the other hand, a dynamical approach is intended to explain which, if any, such equilibria are actually reached by pointing to a mechanism that shows how they can be reached by agents whose cognitive capabilities are bounded, i.e. they do not conform to the omniscient rational agents of economics.

This type of approach is currently being applied to understand different socially relevant issues, such as the emergence of cooperation [17]; where a paper on spatiotemporal chaos [18] originated a huge number of papers on theoretical models [11, 12]. This effort further fructified in several experiments with human subjects [19–21] leading to the understanding of the dynamics in terms of moody conditional cooperation and reinforcement learning [22–24]. In this context, a very pressing issue that is key to understanding human societies and how they can be nudged towards cooperating with each other is that of identity (religious, linguistic, political, etc) as the source or reason for different preferences [25]. Indeed, the interplay between our preferences and the influence of our social relationships (friends, acquaintances, coworkers) on our choices arises in many aspects of our daily life. This occurs, for instance, when we choose friends [26] or neighbors [27], a process where individual preferences are a key in our decisions. Another example of the importance of preference is the large influence peers have on human behavior [7], affecting whether people's behavior aligns to that of their social relationships [28]. Such social influence effects range from which products we buy [29], to the decision to get involved in criminal activities [30], or to our participation in collective action [31]. Particularly important is the case of strategic interaction in networks, a realization of which is the situation in which one has to decide on a technological product that should be compatible with the co-workers's choices. This is a case of a coordination problem, in the class we will discuss below, and clearly choices change depending on others' decisions, but every person has her own initial preference [32]. All these particular situations boil down to a specific research question: what is the effect of individual preferences on strategic interaction, be it of the coordination or anti-coordination type?

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We study this issue in a very broad range of socially relevant scenarios, by using the model introduced by Hernández *et al* [33]. This is a generalization of an earlier work [29] where two entire classes of games were studied, namely coordination games and social dilemmas (more precisely, strategic complements and strategic substitutes) in random networks. In [33], the problem of diversity in preferences was analyzed by considering that there are two different types of players in the population, and that each type prefers (because the corresponding payoff is larger) one of the two available actions. Therefore, coordination and/or cooperation becomes more difficult, in so far that agents have incentives to choose a specific action that yields more benefit to them irrespective of the choices of those with whom they interact. In fact, as has been recently shown [34], this difficulty in coordination is predicted to be largely dependent on the payoff ratio between the preferred and the disliked actions, but it may even disappear when payoffs become similar. In this context, we here address a number of issues that are relevant from viewpoints of both the evolutionary dynamics of complex systems and its application to societal issues. Firstly, we intend to identify the effect of the presence of agents with different preferences in the system and how this effect depends on the network structure. Secondly, we want to understand whether these effects change in cases where all agents would prefer to choose the same action as the rest (complements/coordination) or different actions (substitutes/anti-coordination). Last, but not least, we want to assess the relevance and validity of our approach by comparing with existing experimental results.

Our results are organized as follows: we begin by introducing the main game theory, focusing on the conceptual differences that the preferences paradigm brings in to the homogeneous framework (section 2). We then study in section 3 the dynamically relevant equilibria and compare them with the ones found in [33, 34] with an analytical, static approach. Subsequently, we also extend the study to the case of a scale-free network and proportion imitation, comparing the differences when the same games are played in the absence of individual preferences. Our next step is to look into the case of incomplete information (section 4), where agents do not have knowledge about their neighborhood, which we analyze having the complete information situation case as our reference point. In this case, only best response case is used because, as we will discuss below, proportional imitation cannot be applied for lack of information. In the conclusion (section 5) we summarize our results, compare with the available experimental results, and discuss how they give insight on solving coordination problems in situations of conflicting preferences.

2. Model

The building blocks of our models are a set of agents, a game that specifies their interaction, and a network of connections between them that rules who interacts with whom. Each agent has two possible actions, which we label $X = \{0, 1\}$ and a preference for one of them. Due to this preferential heterogeneity, individuals who choose their preferred action gain greater payoffs than when they choose the other one, for every game in the families we will consider below. This is mathematically represented by two

parameters, which represent the rewards for choosing the liked or disliked option, and hence affect the incentive to change action and so the dynamics of the game. In order to cover an ample set of games (i.e. of possible interactions between people) we work with a very general payoff function

$$u_i(\theta_i, x_i, x_{N_i}) = \lambda_{x_i}^{\theta_i}[1 + \delta \sum_{j\in k_i} I_{\{x_j=x_i\}} + (1-\delta) \sum_{j\in k_i} I_{\{x_j\neq x_i\}}] \tag{1}$$

where x_i is the action taken by agent i, k_i are the neighbors of agent i as specified by the corresponding network, x_{N_i} is the vector of actions taken by i's neighbors, θ_i is agent i's preference (that, as actions, can be 0 or 1), and $I_{\{x_j=x_i\}}$indicates the neighbors who choose the same action as i. As for the payoffs λ takes the value α if the agent takes his liked action or β otherwise, where $0 < \beta < \alpha < 2\beta$, and δ defines the kind of game we are playing: if $\delta = 1$ we are playing a coordination game (CG; the best action is to do as others do), if $\delta = 0$ we are playing anticoordination game (AG; the best action is to do the opposite of what the others do). We note that in economics jargon these game families are usually referred to as strategic complements and strategic substitutes, respectively [29], but in this work we prefer to use the names above as they make it easier for the reader to grasp the actual meaning of the two types of interaction. We also note that the original homogeneous model in [29] is recovered when all players have the same preference. Below, we will in addition differentiate two types of arrangements: the first is a situation of complete information, where every individual knows who his neighbours are and what they do at every round of the game; the second is a situation of incomplete information, where agents know how many neighbors they have and the distribution of preferences in the network, but they do not know what the specific preferences of their neighbors are. In this last case, agents can infer the proportion of neighbors who might prefer action 1 or 0 knowing the preferences distribution and their degree, but they do not know exactly the distribution of actions in their local network. Finally, we need to specify the dynamics we will consider in this model. Following [35], we will use the following two dynamics as representative of the more economics-style (best response) and evolutionary (imitation) choices.

2.1. Best response

Let us call χ_i the number of agent i's neighbors who choose action 1, so the number of neighbors that choose action 0 is $k_i - \chi_i$. As described in [33], from the purely static, theoretical viewpoint in economics, we have two thresholds to compare with χ_i in order to permit to agent i to decide which action to take:

$$\underline{\tau}(k_i) = \left[\frac{\beta}{\alpha+\beta}k_i - \frac{\alpha-\beta}{\alpha+\beta}\right], \tag{2}$$

$$\overline{\tau}(k_i) = \left[\frac{\alpha}{\alpha+\beta}k_i + \frac{\alpha-\beta}{\alpha+\beta}\right]. \tag{3}$$

With these two thresholds, the best response for agent i with preference $\theta_i = 1$ in a CG is given by:

$$x_i = \begin{cases} 1 & \text{iff} \quad \chi \geqslant \underline{\tau}(k_i) \\ 0 & \text{otherwise;} \end{cases} \tag{4}$$

Conversely, the best response for agent i with preference $\theta_i = 0$ is given by:

$$x_i = \begin{cases} 0 & \text{iff} \quad \chi_i \leqslant \overline{\tau}(k_i) \\ 1 & \text{otherwise.} \end{cases} \tag{5}$$

These two options for the CG are simply illustrated in the sketch of figure 1. below.

Similarly, in the case of an AG, agent i will choose the liked action when $\chi_i \leqslant \overline{\tau}(k_i)$ for $\theta_i = 1$ and when $\chi_i \geqslant \underline{\tau}(k_i)$ for $\theta_i = 0$. With these results, the predictions from the analysis in [33] are that equilibria in the network are such that all players coordinate on one action (specialized) or both actions are chosen by different players (hybrid). There are two categories of equilibria, depending on whether all players coordinate in choosing the action they like (satisfactory) or at least one player chooses the disliked option (frustrated). So we have four possible equilibria: (i) satisfactory specialized (SS) where all players coordinate on the same action, which is their preferred choice (so, this can happen only in the homogeneous model, where all agents have the same preference); (ii) frustrated specialized (FS), where all players coordinate on the same action, but at least one of them is choosing his disliked option; (iii) satisfactory hybrid (SH), where all players choose their preferred option but there is at least one player with a preference different from the rest, so that both actions are present; and (iv) frustrated hybrid (FH) which presents both actions and at least one player chooses her disliked option. We will analyze below what happens dynamically, i.e. when the game is repeated for a number of rounds starting from a random initial condition and players choose their action in their next round through myopic best response [36], by deciding their next action as a best response to their neighbors' actions in the previous round.

2.2. Proportional imitation

The second dynamics we will consider in this study consists of the imitation of a neighbor: at each time step a fraction of the agents choose one of their neighbors at random and, if the neighbor's payoff is higher than her payoff, she chooses the neighbor's action for the next time step with a probability given by the difference between their two payoffs, according to

$$P\{\pi \to \pi_i^{(t+1)}\} = \begin{cases} (\pi_j^{(t)} - \pi_i^{(t)})/\phi & \text{if } \pi_j^{(t)} > \pi_i^{(t)}, \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

The reason for considering this dynamics is that it is the evolutionary version of the well-known replicator dynamics [4] that, in the limit of an infinite number of agents, can be shown to converge to the Nash equilibria of the game. However, the approach to the equilibria is different from the best response case: in best-response, all agents try to choose directly the action that would give them the largest payoff given the actions of the others, whereas in imitation dynamics agents have a much smaller cognitive capability and limit themselves to imitate some action that they perceive to yield higher payoffs. Imitation is thus a much more realistic dynamic to represent human (or even

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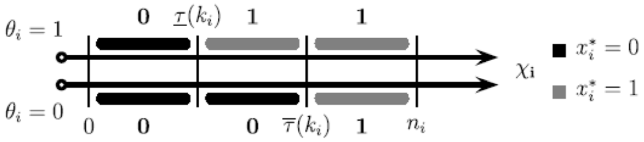

Figure 1. Action change thresholds for both preferences in the Coordination Game.

animal or bacteria) decisions as arising from something akin to a learning (or adaptation) process. On the other hand, the best response is deterministic whereas imitation dynamics is stochastic, which provides another interesting comparison.

2.3. Simulations

In what follows, we report the results of a simulation program in which we have looked at the behavior of the model for its most important parameters: the payoff to choose the liked option, α, the payoff to choose the disliked option, β, and the proportion of 1-preference agents ρ, always respecting the conditions of the games: $0 < \beta < \alpha < 2b$ *and* $1 < \rho < 0$. We have considered networks with $n = 10^2$ nodes (except when specified) and all the results are averaged over a number of simulation iterations each one composed of the amount of time steps t necessary to reach the equilibrium in that specific case. Equilibrium is defined as is customary in this context as a state that once it is reached it is not abandoned by the system. Therefore, for larger values of t we would always observe the same state. Simulations with more nodes have also been made and will be reported below in order to check our conclusions, and the number of iterations has been taken to be 20 or 50, also for verification purposes. Simulations are run over 8×8 different sets of values for α and β, choosing $0.2 \leqslant \alpha \leqslant 0.9$ and $\frac{\alpha}{2} < \beta < \alpha$, and they are as long as needed for the system to equilibrate. We verified our code by checking that, for the homogeneous model, we recovered the results reported in [35], with very satisfactory results. In addition, we considered two types of networks: an Erdös–Reńyi (ER) [37] random graph, for different values of connectivity m, and a Barabási–Albert (BA) [38] scale free graph, with three (the parameter typically referred to as m in the BA model, different from the connectivity m of our ER networks) edges connecting every new node added to the graph to nodes already in the network. The reason for this is that the theoretical predictions summarized above are only valid for an uncorrelated random graph, which is represented by the ER network. Therefore, we find it interesting to include a completely different network, such as the BA one, that is in fact associated to many more realistic social situations.

3. Complete information

We begin by discussing the results where players have complete information about their neighbors's preferences. For the sake of clarity, we present the results separately for each type of game and each type of dynamics. In this section and throughout the paper, in all plots every dot is a particular set of parameters. In equilibria graphics, red dots are the specialized satisfactory equilibria, yellow ones are the specialized frustrated

equilibria, green ones are the hybrid frustrated equilibria, and blue ones are potentially hybrid satisfactory equilibria or hybrid frustrated.

3.1. Coordination game

3.1.1. Best response. In order to compare with the predicted results from the static approach, we discuss first the uncorrelated networks given by the ER model. Our first observation is that, as we raise the connectivity of ER networks, equilibria tend to be more specialized and hybrid equilibria tend to disappear. Figure 2 shows clearly that equilibria are symmetric for different fractions of preferences, as was to be expected as there is nothing intrinsically different between the two types. When the two preferences have an equal number of agents in the population, the final density of agents who choose action 1 in equilibrium takes values in a range around 0.5, a range that tends to decrease the more we raise the ratio α/β. This is due to the particularities of the different networks realized in the simulations, as there may be local environments that, just by chance, make agents choose an action that is not their preferred one.

It is important to realize that the ratio α/β gives us a direct measure of the incentive given to agents to maintain their preferred action instead of changing it, so it is clear that when this incentive is small a larger variety of outcomes are possible. For instance, when $\alpha/\beta \to 1$, if the simulation starts with a 60% of 0 agents, the final equilibria will be a 0-specialized frustrated one, because the 40% of initial 1 agents are not sufficiently motivated to maintain their liked option. On the contrary, when $\alpha/\beta \to 2$, in the same case of a 60% of initial 0 preferences, a relevant part of the 1 agents resist the temptation to go against their preferences, because the incentive to maintain their preferred action is really higher than the one given to change (unless in very specific realizations one 1 agent is surrounded by a large number of 0 agents; this is something that occasionally, but not frequently, will occur). Therefore, the final equilibrium is not specialized anymore, but it is hybrid and there will be less frustration in the final state. This is seen in figure 2 by the fact that for small α/β the transition from one specialized equilibrium in the action of the majority of the agents to the other is much more abrupt than for large α/β, implying that the range of fractions of each type leading to hybrid equilibria is larger in the latter case. This is especially so in the less connected graphs: in the case of $\alpha/\beta \to 2$ in the final states there are more satisfied agents, because they are pushed to maintain their action as they do not have many neighbors who could induce them to change. We can say the opposite in the case with $\alpha/\beta \to 1$ where in the final states we find more frustrated agents and specialized equilibria. Figure 3 confirms this insight by showing the equilibria for the two extreme values of the payoff ration and including the density of frustrated agents. For small α/β, the fraction of frustrated agents grows approximately linearly with the fraction of 0 agents until they reach a 50–50 distribution: this makes sense as for small payoff ratios only a small majority of agents of the opposite preference is needed to make one change. Interestingly, frustration is much lower for larger α/β, reflecting the fact that locally it may pay to keep one's preferred action even if there are more neighbors of the opposite side. This is particularly true for low connectivity networks; larger connectivity leads to a greater chance of having many neighbors of the opposite type forcing one to change (keep in mind that best response is deterministic and always chooses what is best in view of the environment).

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Let us now look at the case of the BA scale free graph. As we can see from the plot, the overall behavior is not far from the ER random graph one with $m = 5$. This is likely to result from the fact that there is a large majority of agents that have a small number of neighbors, and therefore in terms of the total fraction of agents choosing each action this subset of nodes dominates the dynamics. On the contrary, hubs are just one neighbor of other agents, so in a best response environment their contribution to the decision of their neighbors is not particularly relevant. In this manner, we have identified two main variables that determine the type of equilibria that will come out in a coordination game: the connectivity of the graph and the payoff ratio.

In the light of what we know about the homogeneous model [33], we observe that in both models connectivity is a catalyst for the achievement of a specialized equilibrium: the more the connectivity is, the less the hybrid equilibria will be. What was true in the homogeneous case about cooperation (understood as coordination in the Pareto-dominant or more profitable equlibrium) can be also said of the heterogeneous model about coordination: if full cooperation was reached thanks to high connectivity under the same cooperation incentive, now high connectivity allows, under the same reward ratio, to reach full coordination (which means specialized equilibria) in the most of the cases. On the other hand, an important difference with the previous model is that, in the homogeneous case, agents had an incentive to cooperate (α) which helped the achievement of fully cooperative final states. When preference enters the game, there is a payoff ratio which hinders full coordination, because it preserves the satisfaction of the individual. Therefore, preference does qualitatively change the problem and, more importantly, the perception of the outcome of evolution as satisfactory by the individuals.

In order to verify the above conclusions, we raised the number of agents to $n = 10^3$ to see if network size could affect the final equilibria. Comparing graphics in figure 4 with those with $n = 10^2$ nodes we can see that the size of the network fosters coordination. Hybrid equilibria almost disappear, although we find some hybrid equilibria for low connectivities, particularly in ER graphs with $m = 5$ and in BA scale free graphs. Levels of frustration go up since every agent now coordinates better with their neighbors, and this takes the final configuration to a more frequent specialized equilibria. Figure 5 confirms this interpretation: indeed, for $\alpha/\beta = 2$ we observe a small difference from the case with 100 nodes, with a phase transition from a specialized equilibrium to the other that is much sharper for high connectivities. Correspondingly, frustration curves in figure 5 show higher curves than before, since now coordination is more frequent and agents prefer to change action when $\alpha/\beta = 1$. On the contrary, when $\alpha/\beta = 2$ preference matters more when preference distribution is close to equal compositions, and in fact when the distribution is 50–50 the system reaches a more satisfactory equilibrium even if it is not hybrid satisfactory. Therefore, what we observe is that for larger systems coordination is found in a wider range of fractions of 0-preference agents, but that even for 1000 agents there is still quite a sizable range for which hybrid equilibria are possible. Increasing the network size up to 10 000 nodes might have given us a better understanding of the role of connectivity in the reach of equilibria, but this range of sizes was beyond our capabilities due to time consuming computational issues: with 1000 node networks, simulations took up to one day to run all the necessary time steps to reach equilibrium. In any event, by increasing the network size from 100 to

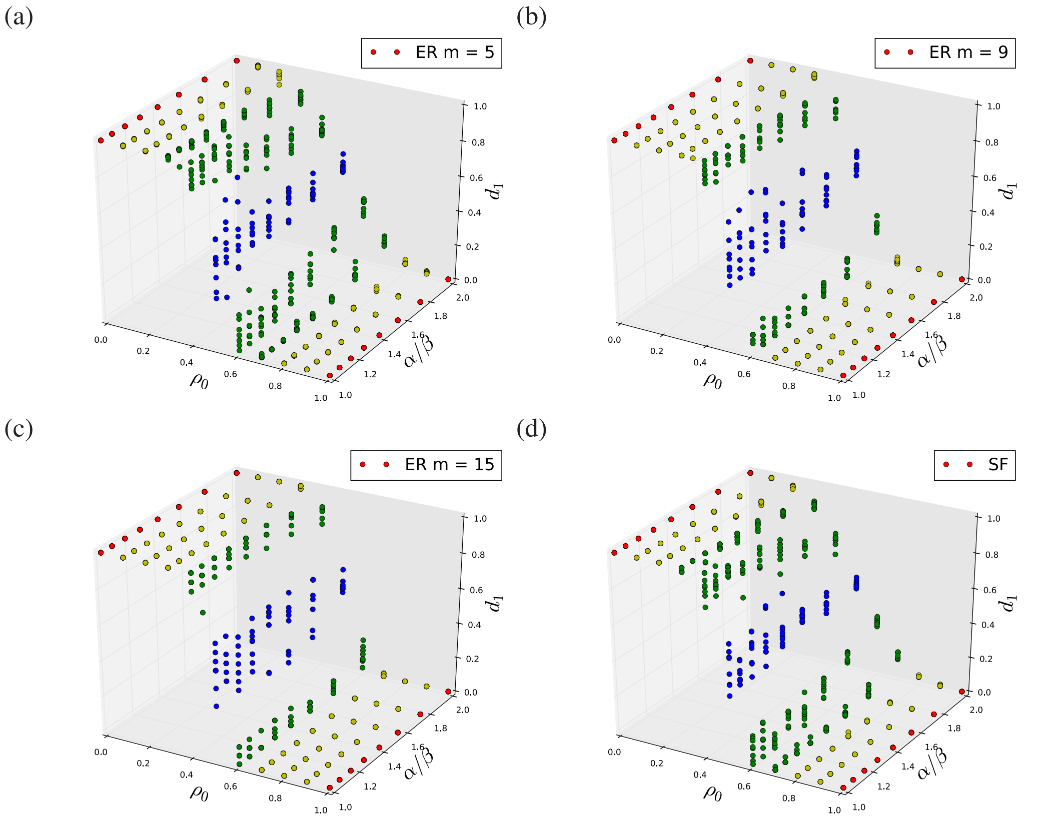

Figure 2. Final average density of agents who play action 1 d_1 against fraction of 0-preference players ρ_0 and reward ratio α/β in equilibrium, for the CG on different ER networks (connectivity as indicated in the plot) and a BA network. Red: specialized satisfactory equilibria; yellow: specialized frustrated equilibria; green, hybrid frustrated equilibria; blue, potentially hybrid satisfactory equilibria or hybrid frustrated.

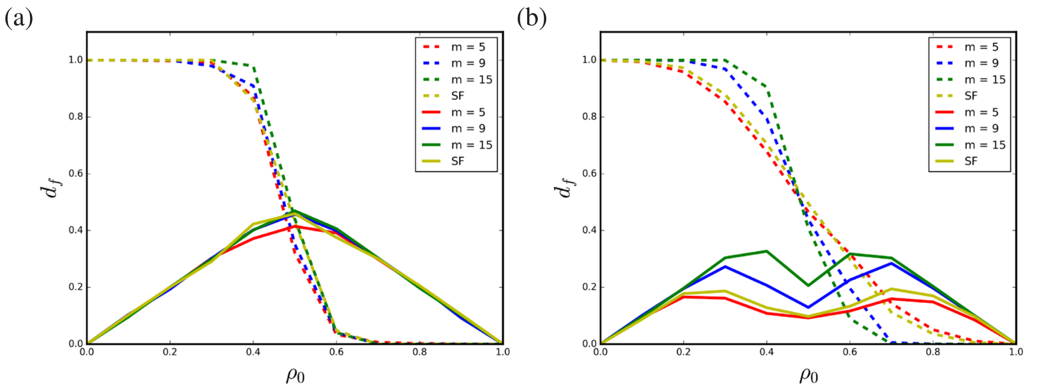

Figure 3. Final average density of frustrated agents d_f over 50 realizations against 0-preference density ρ_0 (solid lines). Shown also is the corresponding final average density d_1 (dashed lines). (a) Coordination game with reward ratio $\alpha/\beta = 1$. (b) Coordination game with reward ratio $\alpha/\beta = 2$. Lines are as indicated in the plot.

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1000 nodes we have already seen an important reduction of the outcomes variability for every kind of network. Indeed, both for ER and BA networks, we did not observe significant variabilities in the outcomes when running different realizations of the same network games with the same conditions. In particular, for BA networks the outcome variablity is much lower than the one observed in ER networks.

3.1.2. Proportional imitation. We now turn to the study of the model under proportional imitation dynamics. In this case, we simulated ER random graphs for different values of connectivities and a BA scale free graph with only 10 iterations to save computing time, because reaching the equilibrium sometimes takes much longer than in the deterministic case of the best response discussed in the previous subsubsection.

It is interesting to keep in mind that in simulations of the homogeneous model with proportional imitation no hybrid equilibria were found [35], the only equilibria arising being specialized. In our study, for the heterogeneous model hybrid equilibria do appear, expecially in the scale free graph as shown in figure 6. While in the ER random graphs hybrid equilibria appear only in the neighbourhood of a 50–50 distribution of preferences, in the scale free graphs almost the whole set of parameters leads to the emergence of hybrid equilibria.

Frustration curves (figure 7) do not show large differences between ER and BA graphs, the main reason being that when the reward ratio is low and the distribution is equal, the selection dynamic goes totally random. This is so because agents choose a random neighbor, independently of their preference, and subsequently they choose their neighbours action if the payoff is better than their own one. When $\alpha/\beta = 1$ this implies that half of the 0-individuals and the half of the 1-individuals will eventually change their action, which results in a 50% of the frustration in the final state. This is independent of the type of network because, for this, dynamics agents update their action without taking into account their whole neighborhood as they only look at a randomly chosen neighbor.

On the other hand, the reward ratio does not affect the sharpness of the crossover from one specialized equilibrium to the other one as the connectivity does: indeed, less connected ER and BA scale free graphs in figure 7 show smoother crossovers for both values of the reward ratio, while the frustration curves are also very similar for $\alpha/\beta = 1$ and $\alpha/\beta = 2$.

3.2. Anticoordination game

3.2.1. Best response. In this subsection, we will be dealing with AG, i.e. strategic interactions in which the best thing to do is the opposite of one's partners. However, this is not easy in so far as in our model players intrinsically prefer a specific action over the other, which may coincide with that of their partners. In our simulations for the AG we do not observe very relevant differences for different connectivities, but in figure 8 we do observe differences between ER and BA scale free graphs: for the former, the dependence on the reward ratio is smoother, but for the BA network (and, to some extent, for the ER network with $m = 5$) it appears that there is a type of behavior when $\alpha/\beta \lesssim 1.5$, and a different one for larger values. Small reward ratios lead to behavior that is mostly independent of the preference composition of the population, whereas

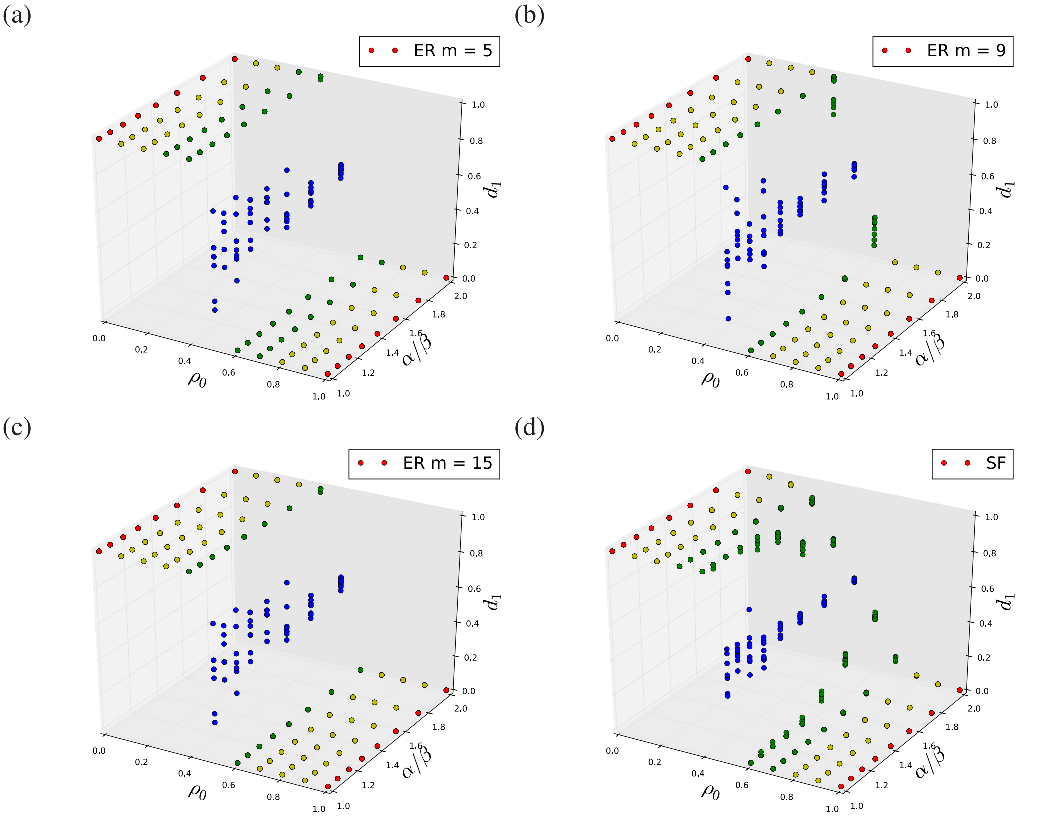

Figure 4. Final average density of agents who play action 1 d_1 against fraction of 0-preference players ρ_0 and reward ratio α/β in equilibrium, for the CG on different ER networks (connectivity as indicated in the plot) and a BA network. Simulations with 1000 agents. Colors as in figure 2.

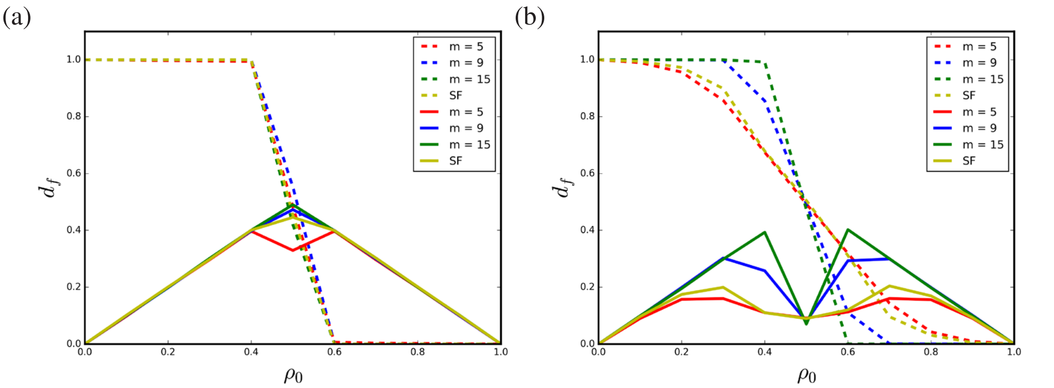

Figure 5. Final average density of frustrated agents d_f over 50 realizations against 0-preference density ρ_0 (solid lines). Shown also is the corresponding final average density d_1 (dashed lines). (a) Coordination game with reward ratio $\alpha/\beta = 1$, (b) Coordination game with reward ratio $\alpha/\beta = 2$.

larger reward ratios give rise to final states in which there is a linear relation between the density of 0 actions and the density of 0 agents. In other words, for large α/β less agents will feel inclined to change their preferred action to anti-coordinate with their neighbors. Figure 9, that shows the frustration dependence on the composition

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for the two extreme cases of the reward ration, indicates clearly that this is the case. Another interesting feature that this plot shows is that, opposite to the case of CG, the minimum frustration occurs for intermediate compositions, being clearer for large α/β. Specialized equilibria do not exist in this case for any population composition, and neither do satisfactory equilibria. Interestingly, for low reward ratios anti-coordination is almost perfect, in the sense that half the agents choose one action and the other half choose the other, but their choices do not correlate with their preferences, which in turn makes half the population frustrated.

Comparing this case with the same one in the homogeneous model is not easy since we had no reward parameters in that case. In the homogeneous model connectivity fostered defection, whereas in the heterogeneous model connectivity has no role as we have just discussed. Similarities can be seen when we take into consideration scale free graphs, because in this structures anticoordination works generally better and frustration is reduced in heterogeneous distributions. In fact, in the homogeneous model, as in the heterogeneous one we see that final configurations are better anticoordinated than those in the random graphs.

3.2.2. Proportional imitation. The dynamics of AG under proportional imitation is, generally speaking, similar to that under best response, but there are some specific features worth discussing. First of all, with a homogeneous distribution the system cannot change its state by definition, since no agent can imitate an action that nobody is playing. This is represented by the extreme cases (red dots) in figure 10. Outside these special values, a large degree of anticoordination is achieved in most cases. In the case of heterogeneos preference distributions with reward ratio $\alpha/\beta \to 1$, anticoordination is reached almost always, except for the scale free networks, where the transition to specialized equilibria is smoother than in the random graphs (see also figure 11). As before, anticoordination works worse when $\alpha/\beta \to 2$, because obviously the agents are more motivated to keep on playing their preferred option. Frustration final values show that for homogeneous distributions there is no frustration in the final states, as of course they did not anticoordinate at all, so they kept on playing their liked action till the end. We observed a difference between the random graphs and the scale free graphs: when $\alpha/\beta \to 1$ frustration is reduced when the distribution is close to 50–50, but not in the scale free graphs where there is a maximum of frustration; when $\alpha/\beta \to 2$ it appears that the scale free architecture makes it more difficult to anticoordinate and to stay satisfied. With $\alpha/\beta = 1$ the crossover to specialized equilibria is sharp, although not so much for low connectivity graphs. On the contrary with a high reward ratio $\alpha/\beta = 2$, connectivity does not affect the equilibria at all, but raising the reward ratio makes the crossover much smoother than before.

4. Incomplete information

Thus far, we have been discussing a situation in which all agents have full information about their surroundings, both about types of partners and about their actions. However, in many social contexts it is difficult to have information about others' preferences, and therefore it is worth considering how the results change when we switch

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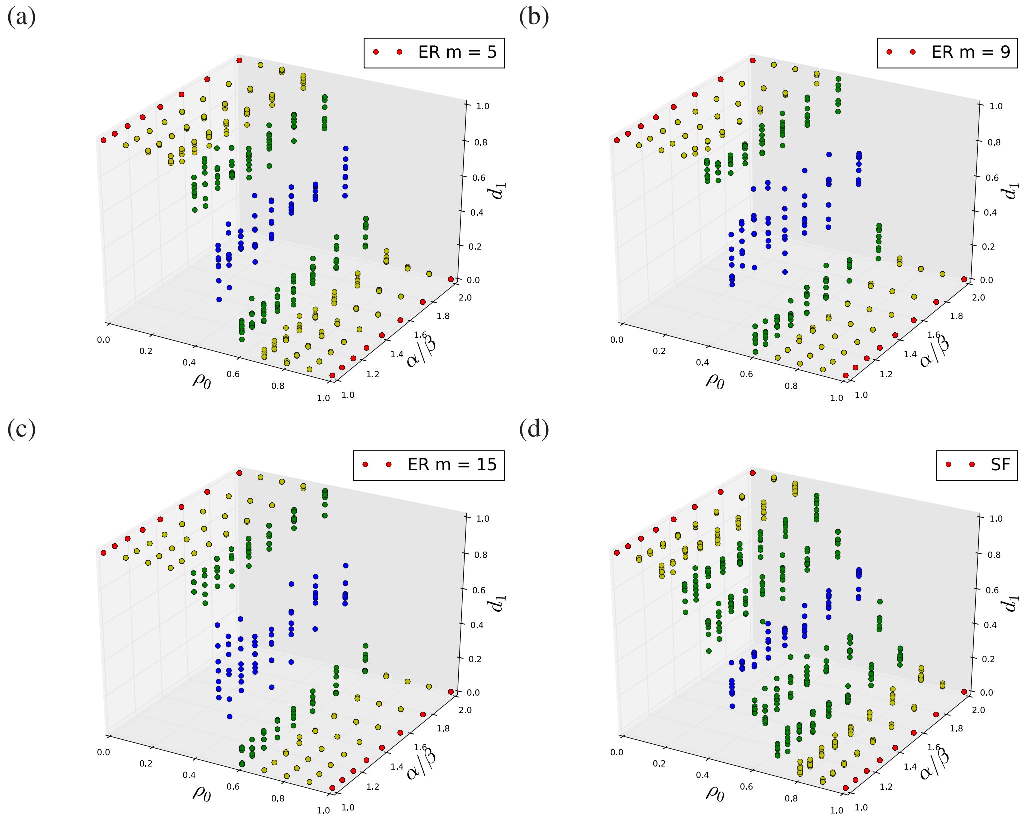

Figure 6. Final average density of agents who play action 1 d_1 against fraction of 0-preference players ρ_0 and reward ratio α/β in equilibrium, for the CG on different ER networks (connectivity as indicated in the plot) and a BA network. Colors as in figure 2.

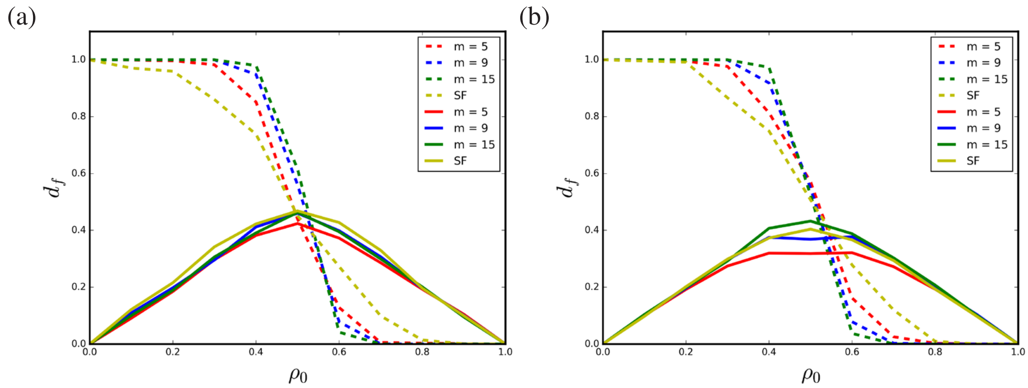

Figure 7. Final average density of frustrated agents d_f over 10 realizations against 0-preference density ρ_0 (solid lines). Shown also is the corresponding final average density d_1 (dashed lines). Coordination game with reward ratio $\alpha/\beta = 1$, Coordination game with reward ratio $\alpha/\beta = 2$.

to an incomplete information framework. In this case, agents know what they like (i.e. their own preference, of course), they know how many neighbors they have, but they do not know who these neighbors are. All they can resort to, to decide on their action, is a distribution of preferences that allows them to estimate the quantity of the two types of

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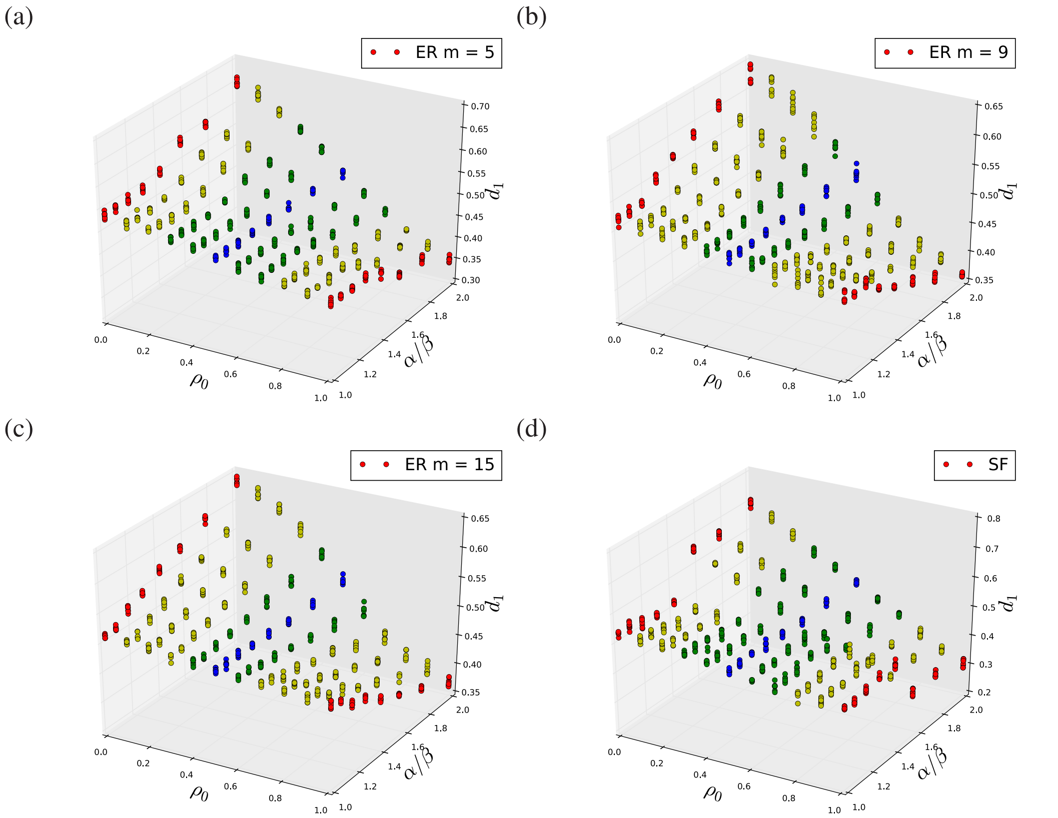

Figure 8. Final average density of agents who play action 1 d_1 against fraction of 0-preference players ρ_0 and reward ratio α/β in equilibrium, for the AG on different ER networks (connectivity as indicated in the plot) and a BA network. Colors as in figure 2.

neighbors they will have around them. This is a quite realistic assumption as very often one has an idea of how opinions or preferences are distributed in the population (e.g. through polls) but is unaware of the specific preferences of the people with whom one is interacting. Our aim is to show if the simulation results of our agents based model fit with the theoretical analysis, which showed how the incomplete information framework reduces the multiplicity of Nash equilibrium with respect to those obtained with the complete information framework.

In what follows, we discuss our results under best response dynamics. In the framework of incomplete information, we cannot consider proportional imitation dynamics, since it is not permitted for the agent to know their neighbour's payoff, the agent knows only their own preference, the number of neighbours and distribution of preferences present in the network. Therefore, a payoff comparison is not possible, making the dynamics unapplicable to this case.

4.1. Coordination game

Compared with the equilibria we found with the complete information framework we see a strongly reduced and ordered set of equilibria in figure 12, confirming what the work of Galeotti *et al* [29] predicted. There are less dots in the figure, indicating that the system ends up with a reduced set of configurations. On the other hand, there

(a) (b)

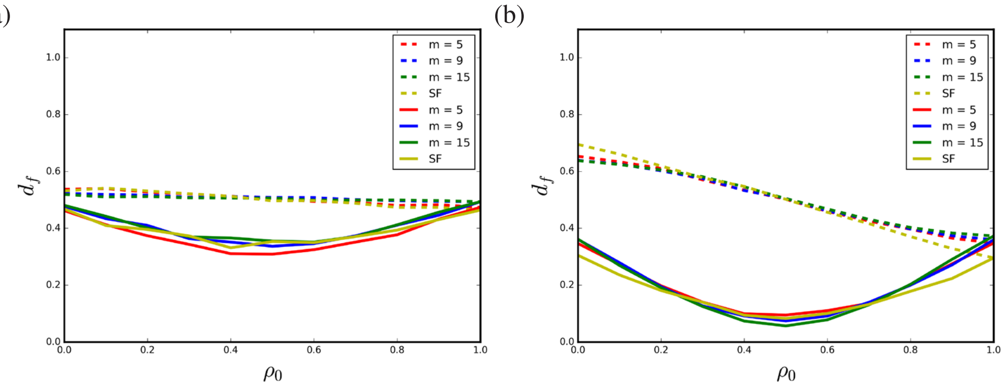

Figure 9. Final average density of frustrated agents d_f over 50 realizations against 0-preference density ρ_0 (solid lines). Shown also is the corresponding final average density d_1 (dashed lines). (a) Anti-coordination game with reward ratio $\alpha/\beta = 1$, (b) Anti-coordination game with reward ratio $\alpha/\beta = 2$.

are similarities between the two informational setups: as we discussed above, raising connectivity implies the loss of many hybrid equilibria, taking the system to more specialized configurations. Looking at frustration in figure 13 we see full satisfactory equilibria when we play games with 50-50 distributions. This agrees with the analytic results obtained [33], where it was found that when the distribution on preferences is very heterogeneous, $\frac{\alpha}{\alpha+\beta} > \rho > \frac{\beta}{\alpha+\beta}$, with ρ being the fraction of players with preference 1 in the population, then satisfactory hybrid configurations appear as a consequence of symmetric equilibrium. The theoretical predictions are in fact more specific, and can be summarized as follows: there exists only a pure symmetric equilibrium, and

- if $\frac{\alpha}{\alpha+\beta} > \rho > \frac{\beta}{\alpha+\beta}$ then every symmetric equilibrium is satisfactory for any connectivity,
- if $\rho > \frac{\alpha}{\alpha+\beta}$ then the action of a given player may only go from 0 to 1 as the degree increases, and all players with preference 1 are satisfied, and
- if $\rho < \frac{\beta}{\alpha+\beta}$ then the action of a given player may only go from 1 to 0 as the degree increases, and all players with preference 0 are satisfied.

As is also shown in figure 13, similarly to the previous cases, with $\alpha/\beta = 1$ connectivity does not affect at all the sharpness of the crossover, but for $\alpha/\beta = 2$ we notice an interesting linear behavior with respect to the proportion of players of one or the other preference. With $\alpha/\beta = 2$, the plot shows that in the range of $0.4 < \rho < 0.6$ full satisfactory hybrid equilibria are reached for the CG in this incomplete information framework. In figure 14 we show the behaviour of frustration in an alternative presentation to better compare the results directly with the theory. As can be see from the four cases, the theoretical limits for ρ to reach full satisfactory equilibria may not be quantitatively correct, but most importantly the analytical prediction demonstrates to be qualitatively correct and even reasonably accurate. Cases (a) and (c) are symmetric, demonstrating the symmetry of the equilibria. In case (d) the full satisfactory

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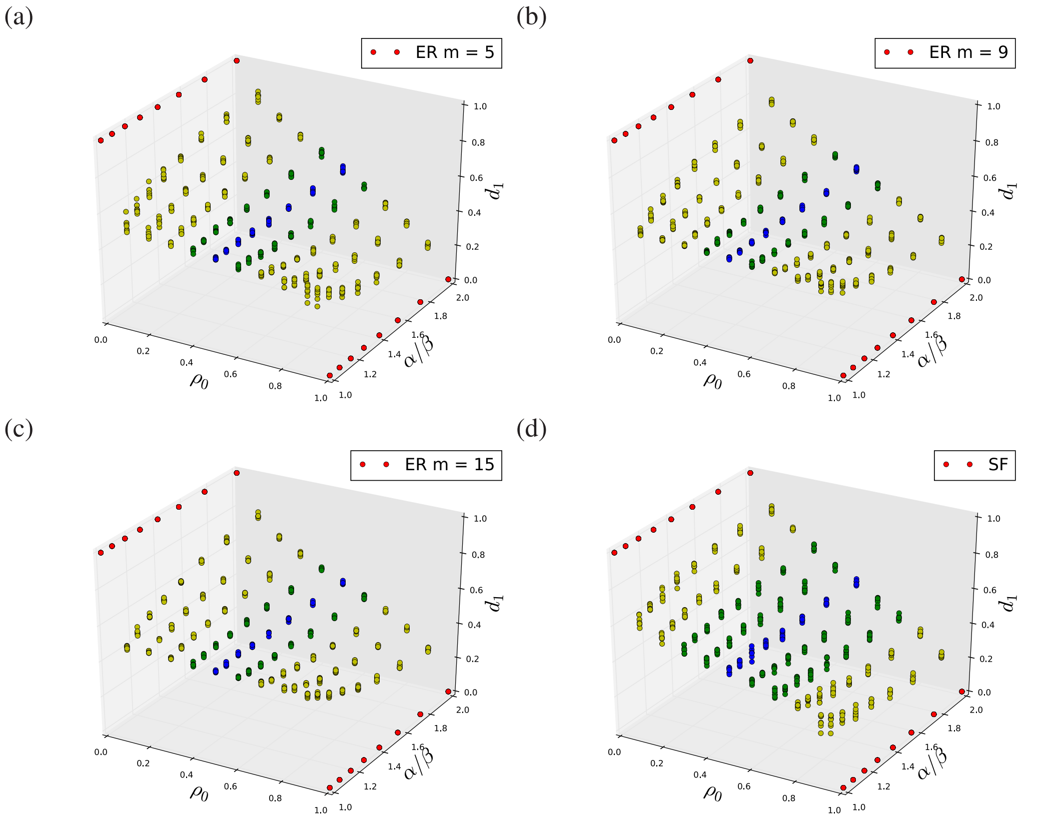

Figure 10. Final average density of agents who play action 1 d_1 against fraction of 0-preference players ρ_0 and reward ratio α/β in equilibrium, for the AG on different ER networks (connectivity as indicated in the plot) and a BA network. Colors as in figure 2.

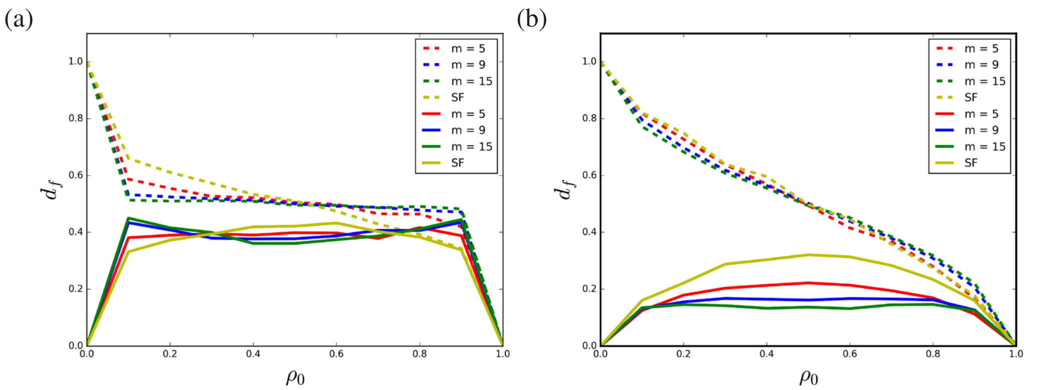

Figure 11. Final average density of frustrated agents d_f over 10 realizations against 0-preference density ρ_0 (solid lines). Shown also is the corresponding final average density d_1 (dashed lines). (a) Anti-coordination game with reward ratio $\alpha/\beta = 1$, (b) Anti-coordination game with reward ratio $\alpha/\beta = 2$.

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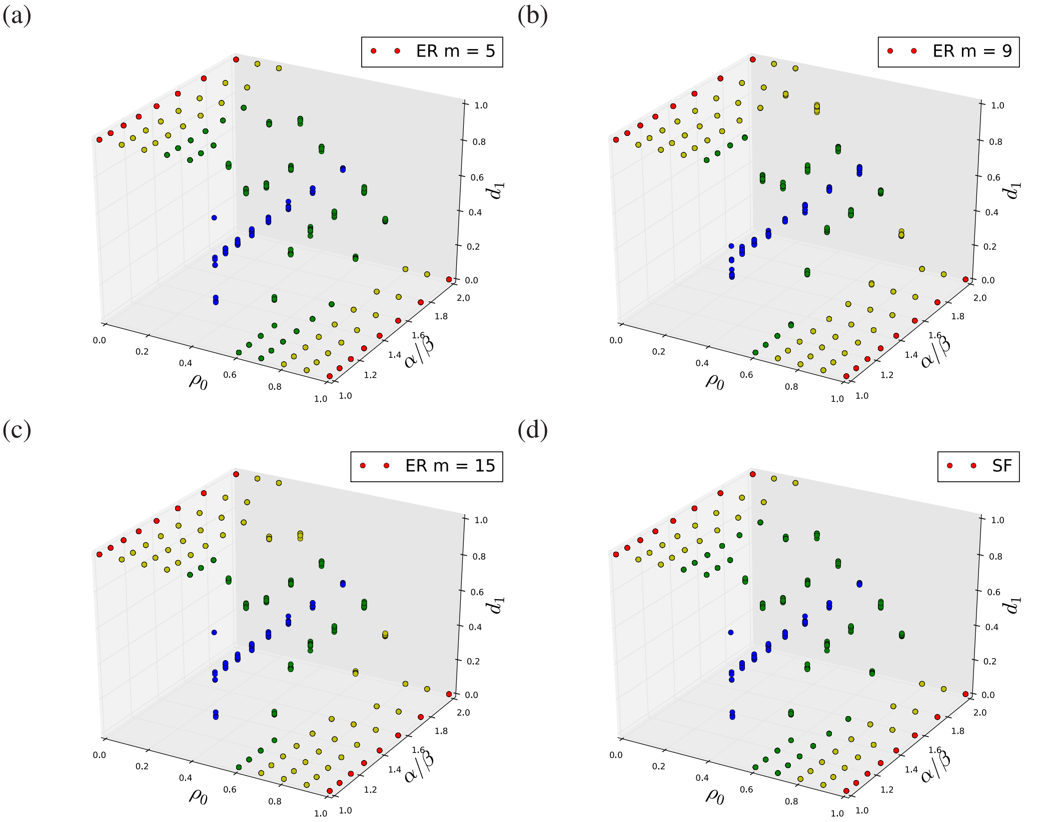

Figure 12. Final average density of agents who play action 1 d_1 against 0-preference density ρ_0 and reward ratio α/β.

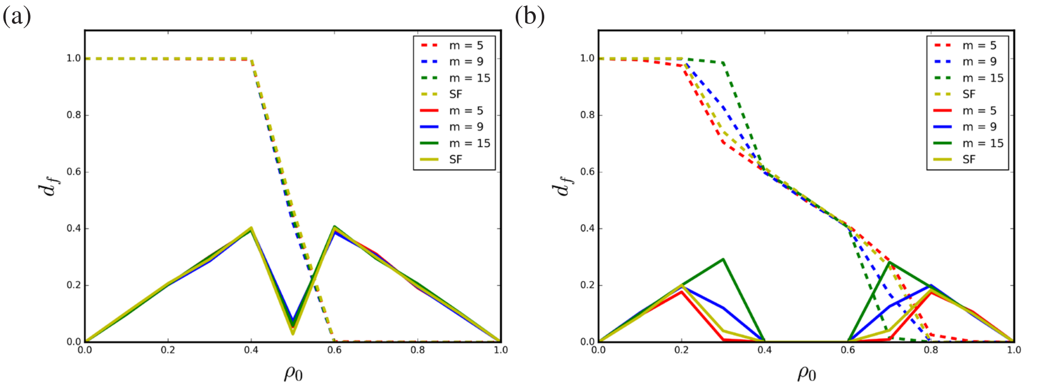

Figure 13. Final average density of frustrated agents d_f over 50 realizations against 0-preference density ρ_0. (a) Coordination game with reward ratio $\alpha/\beta = 1$, (b) Coordination game with reward ratio $\alpha/\beta = 2$.

configuration is never reached, since the 1-preferences density is too low compared to $\frac{\beta}{\alpha+\beta}$, even if it is higher than $\frac{\alpha}{\alpha+\beta}$.

4.2. Anticoordination game

The case of AG under best response dynamics is peculiar because, as discussed in [36], the fact that agents try to anticoordinate leads to unrealistic outcomes when

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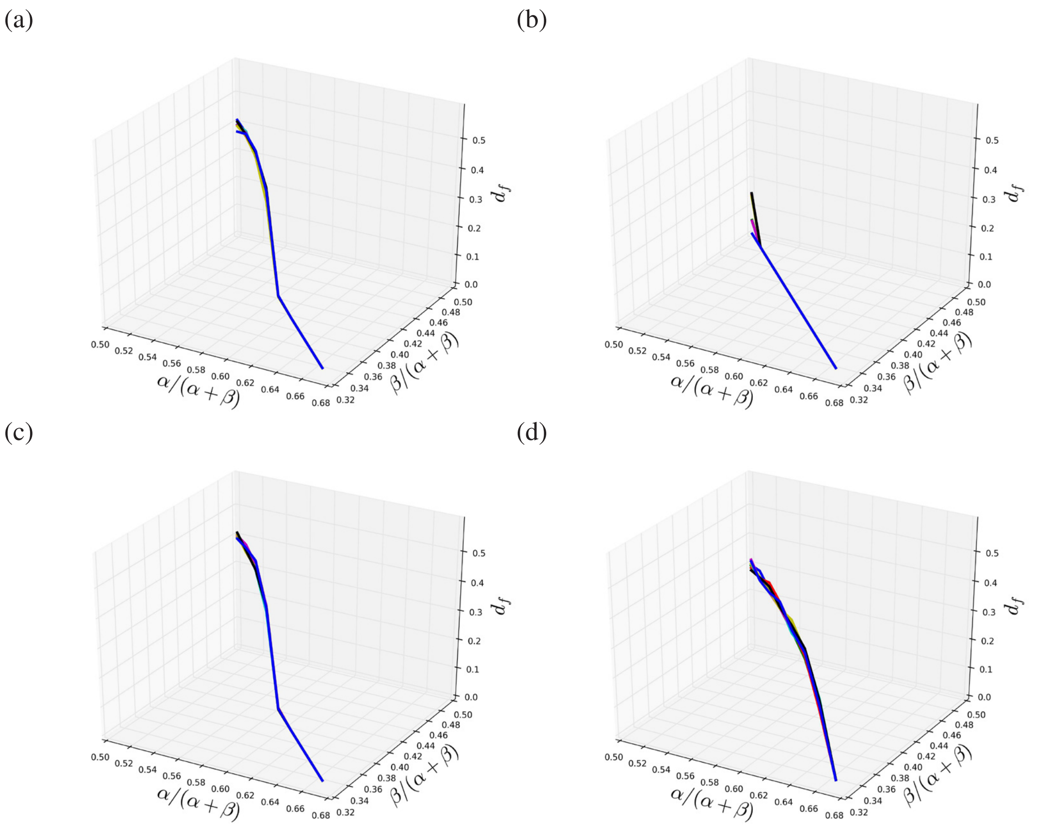

Figure 14. Final density of frustrated agents d_f against reward ratio α/β. (a) $\rho = 0.6$ (b) $\rho = 0.5$ (c) $\rho = 0.4$ (d) $\rho = 0.3$.

the population is homogeneous. Let us keep in mind that agents are given exclusively informations about the distribution of preferences but not about actions, so they act to maximize their payoff expecting that neighbors are going to take their preferred action. In the case of homogeneous distributions, for example when the whole network is made of 1-preferences, every agent knows that he has to anticoordinate with a neighborhood full of 1-preferences, the result is that he will obviously choose action 0, but this happens with every agent in the network. For heterogeneous distributions, the more the connectivity of the graphs, the higher the reward ratio has to be to allow hybrid equilibria to appear in the final configuration, which means that connectivity fosters specialized equilibria, while a large reward ratio, as usual, helps agents to keep satisfied and not change their action. Therefore, anticoordination is reached easier when connectivity is low and reward ratio is high. These conditions for anticoordination lead to strong outcome differences. These same conditions allow satisfactory equilibria to appear. Of course the dynamics shown above for homogeneous distributions give very large values for agents frustration, since the loss of information about neighbors actions makes them totally blind about what is going on. This implies that, while they try to anticoordinate between same preference neighbors, they end up being totally coordinated on the same undesired action. In this sense, it turns out that an equal distribution of preferences is optimal to reach anticoordination, since agents think that half of the neighborhood is like them so they are not pushed to change their action to maximize their payoff. For $\alpha/\beta = 1$, connectivity does not affect at all the dynamics of the

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network, but satisfaction is difficult to achieve because agents are free to change their actions without changing their payoff, and correspondingly there are some outcomes that show full satisfaction when the distribution is close to 50-50, but by no means are all of them satisfactory. On the contrary, when $\alpha/\beta = 2$, frustration is avoided in the most of the cases if the distribution is heterogeneous, and low connectivity helps agent to avoid frustration because they can maintain their liked option. Differing from the same experiment in the complete information framework, here anticoordination is harder to achieve due to the loss of information about the neighbors actions, but satisfactory equilibria appear with some restrictions on reward ratio and connecitivity, which did not appear with complete information.

5. Conclusions

In this paper, we have presented the results of a numerical simulation program addressing the issue of preference in network games from an evolutionary viewpoint. We have considered both coordination and anti-coordination games, as well as different network structures, including random and scale free graphs. We have also studied two dynamics, best response and proportional imitation, which are more economic-like and biological-like, respectively, in order to assess the effects of noise and of a local perspective on decision making. Finally, we completed the picture by looking at two informational contexts, complete and incomplete. This program has allowed us to address the research questions we pointed out in the introduction. Thus, beginning in order of generality, regarding the question about the effects of preferences, a first, general finding is that in all scenarios the heterogeneous model behaves under evolutionary dynamics much closer to the expectations from economic theory [33, 34] than the homogeneous one studied in [35]. Beyond this broad finding, it is important to point out that our model leads to a number of specific predictions which we summarize below.

Let us now summarize our results about the cases of coordination and anticoordination. For the case of coordination games, we have observed that both types of dynamics lead to full coordination for a wide range of compositions of the population. This is in contrast with the homogeneous case, in which the outcome of proportional imitation was always coordination in the risk-dominant, less beneficial action. Here agents tend to coordinate in the action that is preferred by the majority, which leads to a better payoff for the population as a whole, even if the minority is choosing the action they dislike. When there are two preferences in the population, there are only mixed equilibria when the composition is approximately in the range 40%–60% of one type. In turn, this implies that equilibria are never satisfactory, in the sense that for any population composition there will always be frustrated agents playing the action they dislike. This problem aggravates in the already mentioned 40%–60% range, particularly for low α/β values; a higher reward for the preferred action leads to players sticking to their preferences, reducing the degree of coordination, but at the same time lowering global frustration. Connectivity also plays a fundamental role in the achievement of coordination: indeed, more connected networks result in full coordination even in contexts of evenly split population, especially when the reward ratio is kept small,

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i.e. when preferences are not particularly salient. In this respect, we observed that scale free networks with low degree are not connected enough to permit the development of full coordination and a higher density of ties between individuals would be needed to let them achieve higher efficiency. High connectivity has been shown to foster reaching of full coordination: this is a common feature observed in games such as the voter model and the naming game [10]. Interestingly, there are also cases in which high connectivity, in particular the small world property, slows down the reaching of consensus, e.g. for Moran processes [10] or innovation spreading [40]. In these examples high connectivity becomes an obstacle, in terms of speed, to the diffusion of consensus. This behavior may be explained thinking of the increased number of connections, i.e. neighbors, in complex networks with respect to ER networks, which makes necessary an increased number of steps to reach the equilibria. Nonetheless, even if high connectivity slows down the process, full coordination equilibria are found more often than in low connected networks. Thus, it would be interesting to find specific conditions leading to full coordination equilibria in these games more at a faster velocity, which would be important in the context of innovation diffusion. However, the study of the dynamics of the equilibration process is beyond the scope of our work here and is left for future research.

Moving to anti-coordination we have observed that, also for both dynamics, the final states of the model are better in the sense that players do choose the opposite action to their partners. When interaction is of this type, particularly when the reward for choosing the preferred action is large, the amount of frustration is lower than that observed in the coordination problem. This is not what takes place when the reward is small: in that situation, players do anti-coordinate but the action they choose is determined by their surroundings more than by their own preferences, which in the end makes a large fraction of players unsatisfied. It is also interesting that connectivity, while still playing a role, has a less determinant influence on anticoordination than in coordination. As for the dynamics, when there is a large majority of one of the preferences in the population, we have observed that, somewhat counterintuitively, the whole population anticoordinates, as their local updates do not really allow them to realize that they are in fact a majority.

Finally, information is also very important to understand the effect of preference in strategic interactions on networks. When players have only information about the global composition of the population but not of their immediate partners, both coordination and anticoordination become more difficult, except in the extreme cases of a larger majority of one of the preferences or of an evenly split population. Because of the different mechanisms we have discussed within the text, in wide population ranges there are very few frustrated players, and for large reward rations we have even observed many satisfactory hybrid equilibria, i.e. with no frustration whatsoever. Interestingly, we have also found that connectivity is beneficial in this case, as the actions players choose from their knowledge of the global fraction of preferences correspond better to a more populated neighborhood (thus mimicking the behavior of a mean-field model).

In closing, we would like to note that our conclusions point to the soundness of the predictions made from standard economic theory and, therefore, to the applicability of the results we are presenting to real life situations. One particularly appealing conclusion is that, as the economic and biological dynamics yield similar results, our

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findings may have a much wider applicability that purely human societal issues. They may be relevant, for instance, when different strains of a bacteria need to coordinate in producing some chemical. Focusing on the interactions between people, our results are particularly illuminating for the case of coordination, where we have seen that connectivity is beneficial. This indicates that in social situations where preference gives rise to conflict, one possible way to decrease the level of conflict and help people reach consensus is to increase the relations among both communities. Interestingly, recent experiments [39] show that when every player is connected with every other one, even when the population is close to a 50–50 composition full coordination is reached (but not always, some instances of hybrid equilbria have also been observed occasionally). This suggests that in fact the range in which we have found hybrid equilibria may vanish both in the very large size limit and when the network is fully connected. It is important to stress that, in the discussion of the results in [35], up to four economic-style explanations were proposed, only to be discarded because they disagree in one way or another with the experimental results. Therefore, we are providing here a starting point for another approach that can be more fruitful, although its application to the results in [35] in full would require an extension to the case where subjects choose their own links. Similar experiments done on the networks we are studying here, which are amenable with similar laboratory setups, should shed light on the accuracy of our results and confirm or disprove the validity of the evolutionary approach to an economic-like problem. On the other hand, the downside of such a socially efficient outcome is a large minority taking an action they do not like (an issue that might not arise if what is wanted is anticoordination). In this respect, the only way to nudge the population to a better individual situation would be to decrease the saliency of preferences, by making the alternative choice more valuable. We hope that our study encourages more work both on the understanding of the effects of preference in a highly connected work and how to use them to achieve better societal outcomes both at the individual and at the global level.

Acknowledgments

This work was partially supported by the EU through FET-Proactive Project DOLFINS (contract no. 640772) and FET-Open Project IBSEN (contract no. 662725), and by grant FIS2015-64349-P (MINECO, Spain / FEDER, UE).

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