Economics Letters 155 (2017) 154–156

Contents lists available at ScienceDirect

Economics Letters

journal homepage: www.elsevier.com/locate/ecolet

Equilibrium characterization of networks under conflicting preferences

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HIGHLIGHTS

- Characterization of equilibrium in a network when players have conflicting preferences.
- The stronger individual preferences the harder to achieve coordination in choices.
- When the payoff ratio is less extreme, full coordination is always an equilibrium.
- When the level of conflict is low, players choosing what they prefer is not an equilibrium.

ARTICLE INFO

Article history:
Received 29 June 2016
Received in revised form 21 November 2016
Accepted 2 December 2016
Available online 5 December 2016

JEL classification:
C72
C62
D85

Keywords:
Coordination games
Strategic complementarities
Consensus
Networks

ABSTRACT

In this work we characterize equilibrium introduced in configurations for networks with conflicting preferences. We use the model Hernández et al. (2013) to study the effect of three main factors: the strength of individual preferences, the level of integration in the network, and the intensity of conflict in the population. Our aim is to understand how likely is it that social outcomes are either those in which preferences dominate choices or those in which some individuals sacrifice their preferences to achieve consensus with others. Our results show that, the stronger individual preferences, the harder to achieve consensus in choices. However, in cases where the payoff ratio is less extreme, full coordination (consensus) is always an equilibrium. Finally, if the level of conflict is low, individual preferences become less relevant and all players choosing what they prefer is not an equilibrium anymore.

1. Introduction

In this paper, we study situations of conflict where individuals have preferences over a set of choices but these are not always aligned with the preferences of those they relate to, i.e., their local network (Hernández et al., 2013; Ellwardt et al., 2016). These problems belong in the wide class of coordination games with strategic complementarities (López-Pintado, 2006; Zandt and Vives, 2007; Galeotti et al., 2010; Cimini et al., 2015). In such settings, the more neighbors an individual coordinates with, the greater his payoffs are.

Work on this problem has shown that the most salient social outcomes are either those in which preferences dominate choices or those in which some individuals sacrifice their preferences to achieve consensus with others. A full characterization of these equilibrium outcomes is greatly needed to understand the conditions under which the negative effects of conflict may be lessened. Nonetheless, the only approach to this problem, so far, has been to model individual best responses (Hernández et al., 2013). In that paper, the authors describe how players decide depending on their neighbors actions, whereas the aim of this paper is to characterize equilibrium outcomes for an ample collection of networks.

In the following, we consider different types of networks according to three main factors involved in this kind of situations. First, we study variations in the strength of individual

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http://dx.doi.org/10.1016/j.econlet.2016.12.004
0165-1765/

preferences. This is measured through the relation between the payoff of choosing according to one's preference and that of choosing against it to coordinate with others. The second, defined as the level of integration in the network, is the individual connectivity of key players in the network (i.e. the least or the most connected player). The last relevant factor is the intensity of the conflict in the population, arising by the distribution of players with different preferences (i.e., the relative size of the minority against majority).

Our results show that the stronger the individual preferences the harder to achieve consensus in choices. This is due to the fact that the payoffs for choosing what one prefers allow players to disregard the behavior of their neighbors. For smaller payoff differences, consensus arises as a more salient outcome. Moreover, if the level of conflict is low, so that the minority is very small compared to the size of the majority, individual preferences become less relevant and all players choosing what they prefer is not an equilibrium anymore.

2. The model

To develop the equilibrium characterization of the problem of conflicting preferences, we resort to the model introduced in Hernández et al. (2013). In the model, a set of players $N = \{1, \ldots, n\}$ are connected through a network $\{g\}$, where N_i denotes player i's set of neighbors and has cardinality k_i. Each player i has a type $\theta_i \in \{0, 1\}$ and chooses an action $x_i \in \{0, 1\}$. The network game Γ is expressed through the following linear payoff function:

$$u_i(\theta_i, x_i, x_{-i}) = \lambda_{x_i}^{\theta_i} \left(1 + \sum_{j \in N_i} I_{\{x_j = x_i\}} \right) \quad (1)$$

where $\lambda_{x_i}^{\theta_i} = \alpha$ if $\theta_i = x_i$ and β otherwise, and $\alpha > \beta > 0$.

In Hernández et al. (2013), the best response characterization was found in terms of threshold functions, allowing to determine the tipping point where players switch from their liked to their disliked behavior. The threshold value is the minimum fraction of neighbors necessary to coordinate with to guarantee that choosing the preferred behavior gives greater payoffs.

Fixing $\{g\}$, a unilateral deviation by player i changes her choice x_i to choice x_i', where $x_i \neq x_i'$. When no player has incentives to deviate from an action profile $(x_1^*, \ldots, x_n^*)$, that action profile is a Nash equilibrium. Formally:

$$u_i(\theta_i, x_1^*, \ldots, x_i^*, \ldots, x_n^*) \geq u_i(\theta_i, x_1^*, \ldots, x_i', \ldots, x_n^*)$$
$$\forall x_i' \neq x_i^*, \quad \forall i \in N.$$

In our equilibrium characterization, we maintain the labels used in Hernández et al. (2013) to denote the two main equilibrium configurations. If all players in a network choose the action they prefer, this is denoted as a *Satisfactory Hybrid equilibrium* (S_H). Thus, if on the other hand, all players choose the same action, so that some are deciding what they dislike to coordinate with others, it is denoted as a *Frustrated Specialized equilibrium* (F_S). Note that the words "satisfactory" and "frustrated" are used to describe whether all players chose their preferred action or not, and do not imply any particular utility ordering. In fact, it might be the case that a "frustrated" individual has higher utility, than a "satisfactory" individual.[1]

[1] We thank an anonymous reviewer for pointing out the differences in this concepts to help clarify the text.

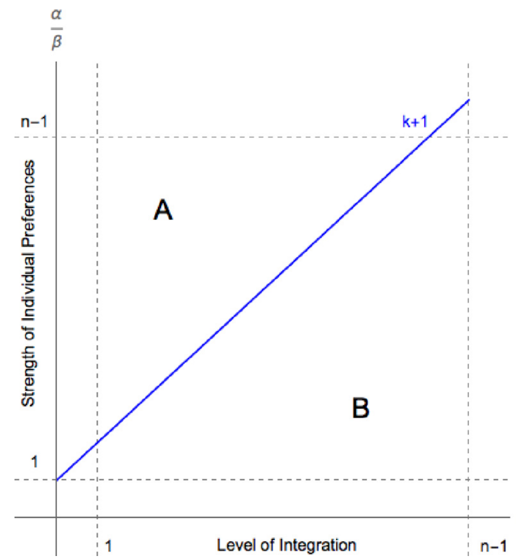

Fig. 1. Regions in the space of strength of individual preferences ($\frac{\alpha}{\beta}$) and level of integration in the network (k) where different equilibria arise.

3. Characterization

In this next Section, we characterize under which conditions these equilibrium outcomes may emerge. We focus on the strength of individual preferences ($\frac{\alpha}{\beta}$) and on the level of integration in the network expressed through player's individual connectivity (k_i), depicted in Fig. 1's y-axis and x-axis respectively.

Let us define the maximum degree in the network as $\overline{k} = \max\{k_i\}_{i \in N}$ and the minimum as $\underline{k} = min\{k_i\}_{i \in N}$. The intensity of conflict is measured through the distribution of types $\pi = (\pi_0, \pi_1)$, in which we assume that there is heterogeneity (i.e. $\pi_0 = \frac{|\{i:\theta_i=0\}|}{n} > 0$ and $\pi_1 = \frac{|\{i:\theta_i=1\}|}{n} > 0$). Note that conflict is highest when $\pi_0 = \pi_1 = \frac{1}{2}$.

Our first result shows that, in cases where the payoff of the preferred action is large enough with respect to the players' individual connectivity, only F_S equilibrium configurations emerge. This outcome arises because every player chooses what she likes, no matter how many connections she has or whom she is connected to. The next Lemma formally states this result:

Lemma 1. *Assume $\overline{k} < \frac{\alpha-\beta}{\beta}$. The action $x_i^* = \theta_i$ for any agent i is a dominant action. Therefore the action profile $\{x_i^* = \theta_i\}_{\forall i \in N}$ is the unique Nash equilibrium in Γ.*

Proof. We prove that player i of type $\theta_i = 0$ has a higher payoff playing $\{x_i^* = \theta_i\}$ no matter the opponents do (a similar proof applies to the case of a player i of type $\theta_i = 1$): $u_i(1, x_j^* | \theta_i = 0) = \beta(k_i - \sum_{j \in N_i} I_{x_j=0} + 1) \leq \beta(\overline{k} + 1) < \beta(\frac{\alpha-\beta}{\beta} + 1) = \alpha < \alpha(\sum_{j \in N_i} I_{x_j=1} + 1) = u_i(0, x_j^* | \theta_i = 0)$. The r.h.s of the first inequality is related to the situation in which a player with $\overline{k}$ connections in the network is connected only to players of the opposite type. The second inequality comes from $\overline{k} < \frac{\alpha-\beta}{\beta}$, the assumption of the Lemma. Since for any player his/her dominant action is to play the same than his/her type, then there exists a unique equilibrium $\{x_i^* = \theta_i\}_{i \in N}$. □

Lemma 1 implies that action profiles $x^* = (0, \ldots, 0)$ and $x^* = (1, \ldots, 1)$ (where every player chooses the same action) are no equilibria in $\Gamma_\pi^{\alpha,\beta}$.

Fig. 1 depicts this threshold as the straight blue line, $f(k_i) = k_i + 1$. Region A is the outcome described in Lemma 1, where the *Satisfactory Hybrid* configurations are the only Nash Equilibria.

Next, we show that, in cases where the payoff for the individually preferred action is not large enough with respect to the players' connectivity, F_S equilibria can emerge (region B of Fig. 1). It is possible to achieve consensus because the payoff relation allows for configurations where players in the minority earn more by sacrificing their individual preferences to coordinate with their neighbors. The following Lemma expresses it formally:

Lemma 2. *Assume* $\underline{k} > \frac{\alpha-\beta}{\beta}$. *Then for all generic distribution* π, *the action profiles* $\{x_i^* = 0\}_{i\in N}$ *or* $\{x_i^* = 1\}_{i\in N}$ *are Nash equilibria in* Γ.

Proof. For the F_S equilibria to emerge, the following inequality must hold (or an equivalent one when player i has type $\theta_i = 0$): $u_i(0, x^* = (0,\dots,0)|\theta_i = 1) = \beta(k_i + 1) \geq \beta(\underline{k} + 1) > \beta(\frac{\alpha-\beta}{\beta} + 1) = \alpha = u_i(1, x^* = (0,\dots,0)|\theta_i = 1)$. □

In order to further characterize equilibrium configurations, we need to consider variations in the intensity of conflict. This is measured through different distributions of the players' types (π). Our last result states that, if the conflict is not intense, given that the relative size of the minority is sufficiently small, there are no S_H equilibria. If there are not enough players of the minority type for the player(s) with the smallest degree ($\underline{k}$) to choose their preferred action, their best response is to choose their non-preferred one. As a consequence, S_H equilibria can be ruled out.

Lemma 3. *Assume* $\underline{k} > \frac{\alpha-\beta}{\beta}$. *If* $\min\{\pi_o, \pi_1\} \cdot n \leq \frac{\beta(\underline{k}+1)-\alpha}{\alpha+\beta} - 1$ *then the action profile* $\{x_i^* = \theta_i\}_{i\in N}$ *is not an equilibrium profile in* $\Gamma_\pi^{\alpha,\beta}$.

Proof. If there exists one player for whom it is profitable to play a different action than $x_i^* = \theta_i$ the Lemma holds. Let i be a node with $\underline{k}$ neighbors and type 0, where type 0 corresponds to the minority (this proof holds when the player has type 1 and belongs to the minority). Then, $u_i(x_i^* = 0, x_j^* = \theta_j|\theta_i = 0) = \alpha(\sum_{j\in N_i} I_{\theta_j=0} + 1) \leq \alpha\left(\frac{\beta(\underline{k}+1)-\alpha}{\alpha+\beta}\right) = \frac{\alpha\beta\underline{k}+\alpha\beta}{\alpha+\beta} - \frac{\alpha^2}{\alpha+\beta} < \frac{\alpha\beta\underline{k}+\alpha\beta}{\alpha+\beta} - \frac{\beta^2}{\alpha+\beta} = \beta\left(\underline{k} - \frac{\beta(\underline{k}+1)-\alpha}{\alpha+\beta}\right) = u_i(x_i^* = 1, x_j^* = \theta_j|\theta_i = 0)$. The first inequality comes from the assumption of the Lemma, and its r.h.s is the case where all players of the minority are connected to player i. The second inequality introduces in its r.h.s the utility for player i, when she chooses her disliked action. In this case she earns a higher payoff. □

4. Conclusions

Our aim in this paper has been to characterize equilibrium configurations in networks with conflicting preferences. We have focused on finding the conditions under which the most salient equilibrium configurations, as defined in the existing literature (Hernández et al., 2013), emerge.

Our results show that in settings where individuals have large incentives to pursue their preferences (compared to the payoffs of choosing what they dislike), it is very unlikely to achieve social consensus. Therefore, there are no Frustrated Specialized equilibria. However, in cases where the payoff relation is less extreme, full coordination (consensus) is always an equilibrium. In those cases, players of a given type are willing to sacrifice their preferences in pro of a higher level of coordinations with their neighbors. This points to a key aspect of social conflict: when there are mechanisms that attenuate the salience of individual preferences, consensus is more likely to be achieved. Additionally, if the minority is very small compared to the size of the majority, individual preferences lose strength and the Satisfactory configuration is not an equilibrium anymore.

We would like to conclude with a few words regarding social efficiency. In an homogeneous network, the *Satisfactory Specialized* equilibrium (where all players have the same type and choose their preferred action) is always the socially efficient outcome. In the presence of heterogeneity, if the network is fully connected, the efficient equilibrium is the *Frustrated Specialized* where all players choose the action preferred by the majority. In less connected structures, the efficient configuration depends both on the distribution of types and the players' positions in the network, is out of the scope of our current study.

In further research, we aim to experimentally test under which conditions can the salience of conflict be reduced.

Acknowledgments

This work was supported by Universidad Carlos III de Madrid, MEC Spain (ECO2013-46550-R), the Generalitat Valenciana (PROMETEOII/2014/054) and the European Commission H2020 FET Open RIA 662725.

References

Cimini, G., Castellano, C., Sánchez, A., 2015. Dynamics to equilibrium in network games: individual behavior and global response. PLoS One 10, e0120343.
Ellwardt, L., Hernández, P., Martínez-Cánovas, G., Muñoz-Herrera, M., 2016. Conflict and segregation in networks: An experiment on the interplay between individual preferences and social influence. J. Dyn. Games 3, 191–216.
Galeotti, A., Goyal, S., Jackson, M.O., Yariv, L., Vega-Redondo, F., 2010. Network games. Rev. Econ. Stud. 77, 218–244.
Hernández, P., Muñoz-Herrera, M., Sánchez, A., 2013. Heterogeneous network games. Games Econom. Behav. 79, 56.
López-Pintado, D., 2006. Contagion and coordination in random networks. Internat. J. Game Theory 34, 371–381.
Zandt, T.V., Vives, X., 2007. Monotone equilibria in Bayesian games of strategic complementarities. J. Econom. Theory 134, 339–360.